Improving the Intellectual Quality of Pedagogy in Primary Classrooms through Mathematical Inquiry

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An empirical study was conducted with the aim to develop teachers' confidence and proficiency with teaching mathematics through inquiry. The study followed 41 primary teachers and compared a regular mathematics lesson to a lesson taught using an inquiry approach; 19 of these teachers were also followed over three years. Lessons were coded on the extent of intellectual quality in the lesson across six dimensions. Higher order thinking showed the most gains over time. Implications for research and practice are given.

The aim of this article is to seek patterns in teachers' pedagogies that promote intellectual quality when they teach mathematics through inquiry. With this aim in mind, an empirical study observed aspects of teachers' pedagogical practices that showed evidence of intellectual quality and investigated the following two research questions.

1. What differences are there, if any, between a teachers' inquiry pedagogy and her traditional pedagogy in teaching mathematics?
2. Do different aspects of inquiry pedagogy change as teachers gain experience in teaching mathematics through inquiry over three years?

Reports consistently point to the poor quality of pedagogy in mathematics classrooms (Mills & Goos, 2011; Stigler & Hiebert, 1999). Traditional approaches to mathematics pedagogy tend to focus on reproduction of procedures rather than to challenge students intellectually to think and reason mathematically. While low-level skills are easy to teach and assess, they miss characteristics of learning that are valued such as proposing and challenging mathematical ideas (Goos, 2004). The outcome of this has been poor student valuing of mathematics and declines in those choosing to study mathematics beyond what is compulsory (McPhan, Morony, Pegg, Cooksey, & Lynch, 2008). In addition, even high performing students are not seeking to make sense of mathematics (Schoenfeld, 1991). These shortcomings point to low levels of intellectual quality in mathematics lessons, significantly lower than in other school subjects (Mills & Goos, 2011).

Mathematical inquiry has been argued to promote the intellectual demands desired in mathematics. "Rather than rely on the teacher as an unquestioned authority, students in [inquiry-based] classrooms are expected to propose and defend mathematical ideas and conjectures and to respond thoughtfully to the mathematical arguments of their peers" (Goos, 2004, p. 259). Inquiry has a number of interpretations. In this paper, inquiry is taken as the process of solving ill-structured problems, where the problem statement, goals or solution pathways contain ambiguities that require negotiation. While the ideals of inquiry promote higher quality teaching (Hmelo-Silver, Duncan, & Chinn, 2007), the practicalities have shown to be challenging (Maaß & Artigue, 2013; Makar, 2012).

In this article, we focus on teachers' pedagogies in developing intellectual quality in Australian primary classrooms, as interpreted by the Productive Pedagogies (QSRLS, 2001) framework. Two analyses were conducted. The first, drawing on data from 41 primary teachers in a regular mathematics lesson and initial inquiry lessons, and the second, based on 19 primary teachers over three years.

2016. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia), pp. 431–438. Adelaide: MERGA.

Methods

The data in this paper come from a longitudinal study designed to understand primary teachers' evolving experiences over time with learning to teach mathematics through inquiry. For this paper, the focus was on quantitative data illustrating *intellectual quality*. This section describes the context of the study, participants, data collection and analysis.

Context and Participants

The study took place in three phases over seven years and aimed to understand how primary teachers' experiences and pedagogies evolved as they taught mathematics through inquiry. A fourth unfunded phase has allowed ongoing interactions with interested teachers. The teachers have come from five schools in suburban or rural Queensland Australia. None of the teachers had prior experience teaching mathematics through inquiry. Although the maximum number of teachers in the study at any time was 40 (Phase III), data were collected from 58 different teachers over the study.

In each phase of study, teachers were involved in professional development 2 or 3 days per year. Most professional development sessions consisted of three activities: time for sharing experiences and discussing challenges or insights about teaching mathematics through inquiry; an inquiry problem or related activity for the teachers to engage in as learners and discuss; and time for collaborative planning and sharing. In most cases, professional development days were held with teachers divided by geography or grade level to keep groups relatively small. Teachers articulated that these sessions were highly valuable to them to have time to think, share ideas, validate common experiences and boost their knowledge (Makar, 2007). Observations of lessons also acted as professional development as researcher and teacher could discuss the details of a lesson in progress.

Data Collection and Sampling

The data reported in this article consisted of observations of primary teachers' lessons involving mathematical inquiry. Teachers were also asked to teach one "regular" (traditional) mathematics lesson. In most cases, classes were videotaped to allow for later review; resources in Phase III sometimes limited these to only observation (no video). Each teacher's data were categorised in five categories, mostly based on terms of experience teaching inquiry: (R) regular mathematics lesson; (A) first term of teaching mathematics through inquiry; (B) rest of first year (second, third, fourth term); (C) second year (terms 5-8); and (D) third year (terms 9-12) teaching mathematics through inquiry. If at least two lessons were observed in a term, the term was counted towards a teacher's experience teaching inquiry. Therefore, if a teacher was unavailable or could not contribute a substantive inquiry for a term, her level of experience did not increase that term. In practice, therefore, teachers may have needed more than 4 terms to complete one "year" of experience. This flexibility ensured that teachers remained in the project, kept morale high, and respected periods of leave or unavailability. It also acknowledged the contexts of the schools, one of which had quite high staff turnover. The data were collected so that teachers were not compared to other teachers; teachers were compared to themselves. This was another element that was seen as more respectful of teachers, avoiding an "us" and "them" judgement.

Because of the large amount of data collected over the study (around 1500 lessons, about half from school S1) and imbalance of numbers of lessons for each teacher, the data were sampled from each teacher using stratified random sampling. Normally, two lessons from each teacher were randomly selected in each category of their inquiry experience (A, B, C, D), if video or coded observations existed. In a small number of cases, only one lesson was available, including regular mathematics lessons (R). For the purpose of this article (to answer our research questions), two cross sections of data were sampled. The first consisted of the 41 teachers with data in categories R and A. This allowed general comparison of a teacher's regular mathematics lesson and her first inquiry lessons (no prior experience teaching inquiry). The second cross section consisted of the 19 teachers for whom data were collected into the third year (D). This smaller subset allowed for a comparison of each teacher's pedagogy over time as they gained inquiry experience.

Framework for Data Analysis

Analysing classroom practices is challenging due to their complexities, the contested nature of what is valued in classroom practices and the acknowledgement of a subjective relationship between valued practices and what is observed. The Productive Pedagogies Classroom Observation Scheme (QSRLS, 2001) was selected as a useful approach to describe classroom practices which exhibited characteristics that might be expected in an inquiry classroom. The Productive Pedagogies (PP) framework consisted of 20 dimensions organised around 4 clusters: *Intellectual Quality*, *Supportive Classroom Environment*, *Connectedness* and *Recognition of Difference* (Table 1). One of the main recommendations of the QSRLS (2001) study was the urgency of improving the intellectual demands of classroom pedagogy. Recent large scale research using the (revised) PP framework reported that this problem persists, particularly in mathematics, where "in both primary and secondary schools, mathematics is the least intellectually demanding subject in the way it is taught" (Mills & Goos, 2011, p. 480).

Table 1
Productive Pedagogies (QSRLS, 2001): Four Clusters and their Dimensions

Intellectual Quality	**Supportive Classroom Environment**
Knowledge presented as problematic	Students' direction of activities
Higher order thinking	Social support for student achievement
Depth of knowledge	Academic engagement
Depth of understanding	Explicit quality performance criteria
Substantive conversation	Student self-regulation
Meta-language	Narrative
Connectedness	**Recognition of Difference**
School subject knowledge is integrated	Knowledge explicitly values all cultures
Link to background knowledge	Representation of non-dominant groups
Connectedness to world beyond classroom	Group identities in a learning community
Problem-based curriculum	Active citizenship

The *Intellectual Quality* cluster, the focus of this article, consisted of six dimensions, each one focusing on an aspect of classroom practice critical for the development of engaging students in high quality work (QSRLS, 2001). *Knowledge presented as problematic* highlights knowledge as constructed rather than fixed and that ideas are generated, challenged and even manipulated by humans. *Higher order thinking* values information that goes beyond facts, rules and algorithms and seeks to observe students focusing on reasoning, explaining, generalising to generate meaning. *Depth of knowledge* attends to the quality of knowledge presented by the teacher. It acknowledges knowledge that focuses on big ideas, significant topics and complex concepts. *Depth of understanding*, on the other hand, places the focus on what students are doing. Students demonstrate depth of understanding when they solve non-routine problems, articulate relationships or bring together multiple concepts in decision-making. *Substantive conversation* is visible when students negotiate meaning by building on others' ideas, or create significant conversations around key content or processes. Finally, *meta-language* is observable when the teacher steps back and explicitly discusses disciplinary structure, practices and language.

The Productive Pedagogy Classroom Observation Scheme (QSRLS, 2001) uses a Likert scale to assess each dimension of classroom practice. An example is given in Table 2 for the dimension *Knowledge presented as problematic*. The research team scored the stratified sample of lessons using this framework. We did not see this instrument as an "objective measure" of teaching practices. Applying the instrument promoted extensive and ongoing discussions in the research team about how the descriptors in the items related to mathematical inquiry. Therefore, our aim in using the scale was as an indicator—rather than a "measure"—of pedagogical practices which reflected ideals of mathematical inquiry we valued.

Table 2
Scale for Knowledge Presented as Problematic (QSRLS, 2001)

Scale	Descriptor
1	No knowledge as problematic. All knowledge is presented in an uncritical fashion.
2	Some knowledge seen as problematic - but interpretations linked/reducible to given body of facts.
3	Approximately half knowledge seen as problematic. Multiple interpretations recognised as variations on a stable theme.
4	Explicit valuation of multiple interpretations and constructions of information, presented as having equal status, and being equally accommodated and accepted by others.
5	All knowledge as problematic. Knowledge is seen as socially constructed, with conflicting implications and social functions producing resolution and/or conflict.

Results

In the first section, a comparison is made between teachers' regular mathematics lesson and their first term of inquiry lessons. Their difference score was computed to seek insight into aspects of Intellectual Quality which appeared different between a teacher's traditional and (inexperienced) inquiry-based mathematics teaching. In the second section, a longitudinal analysis observes the subset of teachers who had data for at least 9 terms (into the third year). This allowed a way to seek insight into aspects of Intellectual Quality which appeared to change over time as the teachers gained experience with teaching mathematics through inquiry. These two analyses provided a richer perspective of inquiry teaching from different vantage points.

Regular Mathematics and Mathematical Inquiry

Two categories of lessons were assessed across the dimensions of Intellectual Quality. The aim of the analysis was to compare the intellectual quality of teachers' regular mathematics lesson and their *first* term of (in)experience in teaching mathematics through inquiry. Means (sd) are provided in Table 3 for the regular mathematics lesson (R), first term teaching inquiry (A) and the difference RA (= A – R). RA was assessed with a t-test (H_0: RA = 0), with Cohen's effect sizes.

Table 3
Mean (sd) Dimensions of Intellectual Quality in a Regular Mathematics Lesson (R), Early Experiences with Inquiry (A), and their Differences (RA), n = 41.

Intellectual Quality Dimension	Mean (sd) R	Mean (sd) A	Mean (sd) RA	p-value RA	Effect size RA
Knowledge presented as problematic	2.1 (0.67)	3.0 (0.90)	0.95 (0.83)	<0.001***	1.14
Higher order thinking	2.3 (0.82)	3.3 (0.83)	1.01 (1.07)	<0.001***	0.94
Depth of knowledge	2.6 (0.72)	3.1 (0.99)	0.53 (1.08)	0.003**	0.49
Depth of understanding	2.7 (0.66)	3.0 (0.87)	0.29 (1.06)	0.09	
Substantive conversation	1.9 (0.83)	2.8 (0.86)	0.87 (1.26)	<0.001***	0.69
Meta-language	2.2 (0.65)	2.3 (0.70)	0.13 (0.87)	0.33	
Overall	2.3 (0.55)	2.9 (0.69)	0.63 (0.78)	<0.001***	0.81

*p<0.05; **p<0.01; ***p<0.001

The outcome of analysis suggested that overall, the intellectual quality in teachers' initial inquiry lessons was significantly higher than in their regular mathematics lessons. The teachers were inexperienced in inquiry and so this difference in intellectual quality may signify an essential difference in how mathematics is taught in these two approaches rather than a stable improvement in teachers' pedagogy. Within dimensions of Intellectual Quality, four stood out as statistically significant. The greatest difference appeared in how mathematical knowledge is presented. Because mathematical inquiry by definition contains ambiguities that require negotiation, it may support students to see mathematics as a human endeavour (Lakatos, 1976). Higher order thinking was also noted as a dimension with a high effect size. This is important given previous findings that mathematics typically has the lowest levels of intellectual demand of any school subject (Mills & Goos, 2011). Although the depth of mathematical knowledge taught in inquiry lessons was significantly higher than in regular mathematics lessons, this did not extend to a significantly higher depth of student understanding. Understanding is generally built over time and students were also inexperienced with inquiry. Meta-language as non-significant may have indicated that teachers did not yet have an understanding of the language and structures of mathematics in this new context.

Mathematical Inquiry over Three Years

To gain insight into whether and how the intellectual quality of teachers' pedagogical practices changed as they gained experience teaching mathematics through inquiry, teachers' lessons were compared at four junctures over three years (Table 4): their first term (A), the rest of the first year (B), the second year (C) and the third year (D). The overall change (AD = D - A) was calculated for each teacher as well, with this difference tested against the null hypothesis that no change occurred (H_0: AD = 0).

Table 4

Mean (sd) Dimensions of Intellectual Quality in A, B, C, D, and Difference (AD), n=19

Dimension of Intellectual Quality	Mean (sd) A	Mean (sd) B	Mean (sd) C	Mean (sd) D	Mean (sd) AD	p-value DA	Effect size DA
Knowledge presented as problematic	2.9 (0.99)	3.0 (0.75)	3.3 (0.60)	3.8 (0.58)	0.93 (1.12)	<0.001***	0.83
Higher order thinking	3.2 (0.65)	3.3 (0.65)	3.5 (0.78)	3.9 (0.60)	0.72 (0.69)	<0.001***	1.05
Depth of knowledge	3.2 (0.84)	3.3 (0.82)	3.5 (0.86)	4.1 (0.72)	0.87 (0.88)	0.012*	0.99
Depth of understanding	3.0 (0.71)	2.9 (0.74)	3.3 (0.59)	3.4 (0.55)	0.41 (0.70)	0.28	
Substantive conversation	2.8 (0.74)	2.8 (0.83)	3.1 (0.96)	3.2 (0.65)	0.36 (0.81)	0.025*	0.44
Meta-language	2.4 (0.75)	2.5 (0.79)	2.6 (0.78)	2.8 (0.84)	0.42 (1.07)	0.52	
Overall	2.9 (0.55)	3.0 (0.66)	3.2 (0.66)	3.5 (0.50)	0.62 (0.54)	0.002**	1.15

*p<0.05; **p<0.01; ***p<0.001

Overall, the intellectual quality of lessons continued to significantly increase as teachers gained experience teaching mathematics through inquiry. This increase was similar to the increase from teachers' regular mathematics lesson to their first inquiry lessons (Table 3). Similar average ratings of initial inquiry lessons (A) for this smaller subset of teachers (n = 19) and the larger cross-section of 41 teachers with initial data suggests that the teachers who continued into the three years taught at similar levels of intellectual quality to those who only had initial data.

Teachers continued to lack significant improvement in developing students' *Depth of Understanding* and the *Meta-Language* used in class. These data looked at ratings of teachers as they gained experience rather than as students gained experience, however. That is, the teachers did not keep the same students over the three years but began again each year with a new class. Therefore, the lack of improvement in observations of students' *Depth of Understanding* may not be captured well by this instrument as students gain experience with inquiry. Teachers' ongoing difficulty with significantly improving their use of *meta-language* signifies a potential area needing more research and professional development. Using a post-hoc t-test to compare the 19 teachers' regular mathematics lesson with their more experienced teaching of mathematical inquiry suggests that these two dimensions did significantly improve in relation to the more extreme categories from (R) to (D) (change in *Depth of Understanding*: mean = .67, sd = 0.65, p=0.003; change in *Meta-Language*: mean = .57, sd = 0.76, p=0.028), however both of these dimensions of *Intellectual Quality* appeared to be much more challenging to shift.

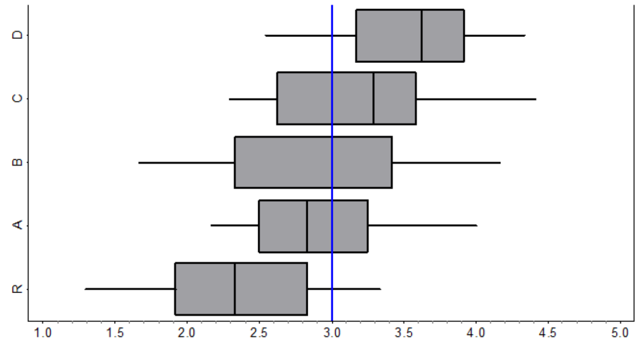

Figure 1: *Comparison of overall average measure of Intellectual Quality from 19 teachers over three years*

Conclusions and Implications

This study examined intellectual quality in lessons involving mathematical inquiry, described by the Productive Pedagogies framework (QSRLS, 2001). In comparing teachers' regular mathematics lessons with their initial lessons teaching mathematical inquiry, four dimensions of intellectual quality showed significant differences—*Knowledge as problematic*, *Higher order thinking*, *Depth of knowledge*, and *Substantive conversation*. This suggests that these aspects of intellectual quality are potentially aligned with the nature of mathematical inquiry. Not that they are necessarily missing from traditional mathematics, but that it is perhaps difficult to teach mathematics through inquiry, for example, without engaging students in conversations and higher order thinking.

The data on teachers' lessons over time continued to show the same dimensions significantly improve as teachers gained experience teaching mathematical inquiry (compared to initial mathematical inquiry lessons). The ongoing improvement of these dimensions could suggest that these are areas that teachers embrace and were possibly not initially very fluent with these areas of practice. It may, therefore, say more about the poor quality of regular mathematics lessons. For example, the higher order thinking dimension had a mean of 2.3 (on a scale of 1 to 5) for a regular mathematics lesson. Most regular mathematics lessons, therefore could be described by the level 2 rank: "Students are primarily engaged in lower order thinking, but at some point they perform higher order thinking as a minor diversion within the lesson" (QSRLS, 2001, p. 6). By the third year of teaching mathematics through inquiry, the mean on higher order thinking was 3.9, implying most lessons could be described typically by the level 4 rank: "Students are engaged in at least one major activity during the lesson in which they perform higher order thinking, and this activity occupies a substantial portion of the lesson and many students are engaged in this portion of the lesson" (ibid).

Knowledge as Problematic and *Higher order thinking* showed the greatest gains in Intellectual Quality as both teachers gained experience teaching mathematical inquiry (AD) and when they first began (RA). These are important areas to improve because of their potential to affect students' understanding of mathematics as a contestable rather than fixed discipline (*Knowledge as Problematic*) and in the potential to improve students' mathematical reasoning through higher order thinking. The other two dimensions which showed improvements in both analyses are also important. The quality and significance of the ideas being taught are at the heart of *Depth of Knowledge*; a mean 2.6 in the regular mathematics lessons signals that there may be issues of low quality content being the focus of most regular mathematics lessons. The depth of knowledge in a regular mathematics lesson, for example, might be improved by spending more time on the "big ideas" in mathematics. Similar arguments can be made for substantive conversation. Not that the conversation in the lessons of experienced inquiry teachers was extremely high (mean 3.2), but perhaps there is an obvious lack of meaningful discussion going on in regular mathematics lessons (mean 1.9).

Two areas not showing significant difference in either analysis—*depth of understanding* and *meta-language*—could signal more challenging areas to generate change. Depth of knowledge presented does not necessarily translate into depth of student understanding, which likely requires longer engagement by *students* in an environment that gives them opportunities to make meaning of mathematics. The low levels of improvement in meta-language also could signal an area teachers are not familiar with.

Acknowledgements

Funding was provided by the Australian Research Council (LP0990184, LP0776703), Education Queensland and the University of Queensland. The author wishes to thank the teachers, research assistants (Janine, Jill, Kym, Lisa, Reshma, and Sue who assisted with coding over the years) and Shelley Dole and Robyn Gillies, grant collaborators 2009-2012.

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